International Journal of Psychology and Psychological Therapy, 2016, 16, 2, 141-155

Relational Frame Theory, Mathematical and Logical Skills: A Multiple Exemplar Training Intervention to Enhance Intellectual Performance

Johan Thirus
Magnus Starbrink
Billy Jansson[*]
Mid Sweden University, Sweden

Abstract

The current study investigated the effects of Relational Frame Theory (RFT) based training on mathematical and logical skills. A sample of 21 Swedish high school students attending first grade and second grade were assigned to either training (n= 10) or no-training conditions (n= 11). Measures of performance on mathematical tests, Raven's Standard Progressive Matrices (SPM), and relational responding tasks were taken prior to and after training. For 8-10 weeks, the experimental group trained using SMART, an online multiple exemplar training program for enhancing relational skills. No significant differences between the groups were found on mathematical performance. A significant increase on SPM performance was observed for the experimental group. The findings are in line with previous research on RFT, suggesting that behaviorally based interventions can enhance intellectual performance. Population characteristics, SMART training procedures, strengths and methodological limitations are discussed.
Key words: RFT, multiple exemplar training, relational responding, mathematical skills, intelligence.

Novelty and Significance

What is already known about the topic?

- Mathematical abilities are assumed to be increasingly important for many aspects of our life.
- Working memory training has been used as a procedure for strengthening mathematical skills..
- Multiple exemplar training is the process of training certain behavioral responses by exposing an organism to a large amount of trials with different stimuli.
- MET improves scores on traditional measures of intelligence.

What this paper adds?

- The current study investigated the effects of multiple exemplar training on mathematical and logical skills.
- This study replicated previous finding showing that multiple exemplar training can increase scores on traditional measures of intelligence.
- This study failed to show that multiple exemplar training is useful for improving mathematical skills.

Scientific and technological achievements of nations are to a large extent dependent on mathematical abilities. Mathematical abilities are assumed to be of great importance with respect to societal development (Butterworth, Varma, & Laruillard, 2011) and longitudinal data show that mathematical skills are the strongest predictor for later school achievement (Duncan, Dowsett, Claessens, *et al*., 2007). The present study aimed to investigate a novel intervention procedure for strengthening such skills.

[*] Correspondence concerning this article: Billy Jansson, Department of Psychology, Mid Sweden University, 831 25, Östersund, Sweden. Email: billy.jansson@miun.se. The first two authors contributed equally to this work.

There is a widespread notion that the components of Baddeley's model of working memory (Hitch & Baddeley, 1976) play an important role in mathematical abilities (DeStefano & LeFevre, 2004). A recent meta-analysis (Melby-Lervåg & Hulme, 2013) showed that even though working memory training tends to show general short-term effects on working memory skills (verbal and visuospatial working memory), the effects do not seem to generalize to other domains, which are believed to be associated with working memory (e.g., mathematical skills). Other researchers have found a significant increase in mathematical reasoning following 6 months of working memory training (Holmes, Gathercole, & Dunning, 2009). However, whether these effects could be attributed solely to working memory training or other contextual factors (e.g., participating in regular curriculum) is not clear.

Strategies based on learning theory and behavior analysis have long been used in educational contexts worldwide and early advances in behavioral science were often applied in these very settings (Kratochwill & Martens, 1994). There are many examples of the successful application of behavior analytic strategies in various school settings, such as behavioral consultation with teachers (Martens & Ardoin, 2002), oral reading interventions (Daly, Garbacz, Olson, Persampieri, & Ni, 2006; Eckert, Ardoin, Daly, & Martens, 2002), school violence and disciplinary problems (Anderson & Kincaid, 2005), classroom management (Meyer, 1999) and homework performance (Miller & Kelley, 1994). Researchers from the behavior analytic position have long claimed that, in order to progress toward more effective methods of teaching various academic skills, a study of the specific behavioral units involved in these skills is needed (Barnes-Holmes, Barnes-Holmes, & Cullinan, 2001).

From a behavioral perspective, both mathematical skills and traditional cognitive concepts (e.g., thinking, language, problem solving, working memory etc.) involve specific behavioral units. However, it is not until recently that the field of behavior analysis has been able to approach cognitions and language from a strictly functional behavioral perspective, thus enabling the empirical study of the specific observable behavioral units that are involved. This endeavor is based, to a large extent, on a behavior analytic theory of human language and cognition called Relational Frame Theory (RFT, Dymond & Roche, 2013; Hayes, Barnes-Holmes, & Roche, 2001).

Even though a functional approach to the study of verbal behavior was attempted by Skinner (1957), Skinner's approach to verbal behavior was mainly theoretical, lacking empirical data. Also, Skinner's definition of verbal behavior failed to distinguish human behavior from the behavior of other organisms, therefore the practical utility of the definition has been questioned (Hayes, Blackledge, & Barnes-Holmes, 2001). Early work on stimulus equivalence conducted by Murray Sidman (1971) showed that humans are able to derive relations (e.g., relations not explicitly taught to the subject) between stimuli. Behavior analysts have since then considered derived relational responding to have important implications for human language and it has become a primary feature of post-Skinnerian research on verbal behavior.

RFT, as a post-Skinnerian account of human language and cognition, has both similar as well as different features compared to Skinner's account (Gross & Fox, 2009). As a behavior analytic theory, RFT has a purely functional focus by emphasizing prediction and control in the study of verbal events. Compared to the structuralistic accounts of human language and cognition, RFT neither emphasizes topography nor assumes inner structures responsible for the development of verbal behavior (Hayes *et al*., 2001). According to RFT, verbal behavior (e.g., human language and cognition) is essentially the behavior of framing stimulus events relationally. Research conducted by Reese (1968, as cited in Hayes & Hayes, 1992) showed that stimuli can be related based on physical properties (non-arbitrary relational responding) by animal species other than humans. However, apart from the physical properties of stimuli, humans also seem to be able to respond relationally to stimulus events according to arbitrary contextual cues, a behavior referred to as "arbitrarily applicable relational responding" (AARR, see Hayes *et al*., 2001, for a review).

The ability of AARR (i.e., relational framing) is a tremendous advantage in the acquisition of new skills, as framing stimuli relationally allows for the occurrence of learning without direct experience with all of the stimuli involved in a learning situation. The behavior of relational framing has three central properties, mutual entailment, combinatorial entailment (also labeled combinatorial mutual entailment) and transformation of stimulus functions. These three properties of relational framing can be considered as separate behavioral units as well as outcomes of relational framing. Understanding verbal behavior as behavioral units is meaningful in that it allows for prediction and control of verbal events (for a thorough discussion of these issues, see Hayes *et al*., 2001).

Mutual entailment involves responding relationally to any two stimuli. For example, if A ☐ B (where ☐ is given the function "same as"), without further training the relation B ☐ A is derived. *Combinatorial entailment* involves three or more stimuli, for example A ☐ B and B ☐ C from which several relations will be derived. From this brief information the relations B ☐ A and C ☐ B are derived and so are the relations A ☐ C and C ☐ A. The ☐ in this example, "same as" (e.g., coordination), is the contextual cue (Crel) that defines the relation rather than topographic properties of the stimuli; hence the relational responding is arbitrarily applicable. A number of different patterns of relational framing have been defined, for example coordination (as above), opposition, hierarchical, comparison, temporal, and spatial framing. For a more thorough presentation of different patterns of relational framing, see for example Hayes *et al*. (2001). When stimuli are framed relationally, the functions of the stimuli are transformed. The concept of *transformation of stimulus functions* refers to the fact that, through framing stimuli relationally, functions of stimuli will change according to the relation that is defined by the Crel and the Cfunc (e.g., the contextual cues that define the stimulus functions involved). In the example above, A will obtain the functions of B and vice versa. If the ☐ in the example represented the Crel "less than", the stimulus functions would have transformed according to that relation, given that the functions of A, B and C have been defined by the Cfunc.

Let us stray from the somewhat technical presentation of RFT and apply the concepts to a couple of everyday educational examples. For the first example, consider a middle school student, Dan, with Swedish as his native language taking English class. In one lesson, the teacher tells the children that she traveled to South America last year to walk in the Amazon jungle. She then tells the children that she had the opportunity to taste a certain kind of fruit called *anercia*. None of the children had ever heard of an *anercia* before. She goes on to tell the children that the *anercia* is exactly like a giant lemon. It is sour in taste with lots of juice inside. She then tells the children that *anercia* is eaten in a special way in South America. The *anercia* is simply cut in half and then the juice is squeezed into the mouth. As the teacher tells this, Dan's mouth starts to salivate and his facial muscles contract.

How is it possible that the previously unheard word *anercia* can have these effects on Dan? According to RFT, his ability of arbitrarily applicable relational responding is the answer. First of all, the Swedish word for lemon is *citron*. A few years ago, when Dan was taught that *citron* is the same as *lemon*, he could quickly derive that *lemon* is also the same as *citron* without any further reinforcement for doing so (e.g., mutual entailment). Mutual entailment is also apparent now that Dan also learnt that *anercia* is the same as *lemon* and hence, he quickly derives that *lemon* is the same as *anercia*. Having learnt that *lemon* and *citron* are the same, and that *anercia* and *lemon* are the same, Dan is also able to derive more relations. For example, that *citron* is the same as *anercia* and that *anercia* is the same as *citron* (e.g., combinatorial mutual entailment). In order to explain the physical effects that the teacher's story has on Dan, we have to consider the third property of relational framing, namely transformation of stimulus functions. For Dan, a *citron* has many stimulus functions (e.g., it's yellow, fairly round in shape, it tastes very sour and so on). Through mutual entailment and combinatorial entailment, these functions transform to the other stimuli (e.g., *lemon* and *anercia*) participating in the apparent relation (Crel), in this case a frame of coordination. However, the actual function that Dan is likely to contact (Cfunc) is not one of shape or color but one of taste because the teacher explicitly talked about eating the *anercia* (as opposed to throwing, smelling, or feeling it). Thus, in conclusion, by learning two relations, Dan is able to derive a total of four other relations without any explicit reinforcement for doing this. Also, while doing this, the functions of the stimuli involved are transformed accordingly.

Now consider the arithmetic concept of *pi*. A child is taught that the symbol π is pronounced *pi* and that *pi* represents the number *3.14*. The teacher then writes the following question on the board: "What do you get if you add 2 to π?". The child correctly answers 5.14. How is this possible? Once again, by explicitly learning two relations (e.g., the symbol π is pronounced *pi*, and that *pi* represents the number *3.14*) the child is able to derive an additional four novel relations: (1) the sound of pronouncing *pi* is the same as the symbol π (mutual entailment), (2) the number *3.14* is the same as the word *pi* (mutual entailment), (3) the symbol π is the same as the number 3.14 (combinatorial mutual entailment) and (4) the number 3.14 is the same as the symbol π (combinatorial mutual entailment).

In the above example, by deriving that π is the same as the number *3.14*, it is easy to see how the child came to the conclusion 5.14. Note once again that the teacher never explicitly teaches this relation. The derived response is controlled by arbitrary aspects of the context (e.g., Crel and Cfunc). As an example of transformation of stimulus functions, note how an arbitrary symbol (π) which previously had no certain function (other than the visual perceptual qualities) in an instant has acquired new functions (such as the auditory perceptual functions which constitute the sound of pronouncing *pi* as well as the numerical value of *3.14*) as a result of the child framing these events relationally in certain ways (e.g., through frames of coordination). From an RFT point of view, arithmetic skills can be operationalized as framing numerical and other symbolic stimulus events relationally.

The behavior of framing stimuli relationally is considered generalized operant behavior and thus cannot be subject to a universal topographic definition. The topography of any kind of relational framing is solely dependent on contextual stimuli, as in the case of lying and imitating. Thus, a purely functional definition is meaningful. Relational framing is acquired early in life and develops through shaping by the elucidation of stimuli and contingent reinforcement by the social context of the individual. Initially, frames of coordination/sameness are explicitly trained, typically by parents who introduce stimuli, such as uttering the word "lamp" and simultaneously pointing at the physical object "lamp" (Stewart & Roche, 2013). Gradually the behavior of relational framing becomes more advanced and vastly expands the behavioral repertoire. Eventually the behavior is to a large extent automated and applied to any stimulus event experienced by the individual. As in the case of other operants, exposure to multiple exemplars is required in order to develop a repertoire of relational skills (Hayes *et al*., 2001).

Multiple exemplar training (MET) is the process of training certain behavioral responses by exposing an organism to a large amount of stimuli over different contexts so that the stimulus control and the behavior is refined (Hayes et al., 2001). Besides shaping by everyday occurring contingencies, MET protocols have been proved to be a feasible intervention in developing relational abilities (Stewart, Tarbox, Roche, & O'Hora, 2013; Cassidy, 2008). In order to assess whether training increases relational abilities (e.g., relational framing skills), a relational ability index (RAI) is typically used. RAI is a measure of performance on tasks of deriving various relations among stimuli. For example, relational abilities can be assessed by recording the number of correct responses, or by a quotient between correct responses and trials needed to complete the test (see for example Cassidy, Roche, & Hayes, 2011; Cassidy, 2008). Strengthening relational framing skills has shown to correlate with more traditionally defined cognitive abilities. For example, O'Hora, Peláez, and Barnes-Holmes (2005) showed that relational ability correlates with WAIS-III subtests arithmetics and vocabulary. Also, Cassidy (2008) showed a mean increase of full scale IQ (FSIQ) of 27.15 points in an intervention group following MET training of derived relational responding, whereas the control group showed a small decline of 2.25 points. A similar study was conducted by Cassidy, Roche, and Hayes (2011) where an automated multiple exemplar relational training procedure was used to train derived relational responding in accordance with *more than*, *less than*, *same as* and *opposite to*. An increase in full scale *IQ* by at least one standard deviation (*SD*) was observed in all participants. Taken together, the results from these studies indicate that relational abilities correlate with traditional measures of intelligence and that a behavioral approach to strengthening relational abilities is potentially meaningful.

Even though MET has shown to be useful in acquiring a range of knowledge using functionally defined stimuli (Fienup, Covey, & Critchfield, 2010; Ninness, Rumph, McCuller, Harrison, Ford, & Ninness, 2005) there is a lack of research investigating the effects of training relational framing on everyday cognitive behavior such as performance on school tasks. Despite numerous studies investigating the effects of multiple exemplar training of relational abilities on cognitive performance, the effects of relational training on intelligence is yet to be assessed by other tests than the Wechsler scales, such as, for example, the Raven's Progressive Matrices. If RFT-based training can increase performance on other validated measures of intelligence than the Wechsler scales, this will further strengthen the validity of using behaviorally based interventions to increase behaviors traditionally conceptualized as cognitive abilities.

Using a between-group design (e.g., training vs. no-training), the main purpose of the current study is to investigate if multiple exemplar training of derived relational responding in accordance with the relational frames *same as*, *opposite to*, *more than* and *less than* (using an automated online training system) will increase mathematical skills in a sample of Swedish high school students. To our knowledge, this is the first study to investigate the effects of training relational framing exclusively on mathematical skills. In addition, another purpose of the present study was to explore the effects of relational training on cognitive abilities using the Raven's Standard Progressive Matrices. The inclusion of Raven's Standard Progressive Matrices was used in order to replicate a previous finding showing that experimentally strengthening relational abilities increase scores on a traditional measure of intelligence (Cassidy *et al*., 2011), but more importantly, to establish that the effect of the experimental intervention (i.e., enhancing relational framing) transfers to other performance tasks, which is a prerequisite for improvement on mathematical performance following multiple exemplar training.

METHOD

Participants, experimenter, and experimental context

Participants were Swedish first grade and second grade high school students in a middle sized town in mid-Sweden. A total of 35 students participated in the study (experiment: n= 18, control: n= 17), 27 female and 8 male. Ages ranged from 16 to 18 years, with the mean age of 17.3 years (SD= .67). Math experience ranged from first to second level in course B (e.g., mathematics for studies in humanistic and esthetics), except for two participants who attended third level of course C (e.g., mathematics for studies in natural science). 16 students attended first grade and 19 students attended second grade. The inclusion criteria used in the current study were: (1) first or second grade high school students, and (2) access to a computer or tablet with Internet connection. No exclusion criteria were employed.

Materials

The independent variable in the current study was the training of arbitrarily applicable relational responding ("relational skills"). The training was conducted using the commercial online training program *Strengthening Mental Abilities with Relational Training* (SMART, see www.raiseyouriq.com). The SMART training program is based on multiple exemplar training (MET), which has been shown to be an effective method in raising relational skills (Cassidy, 2008; Cassidy, Roche, & O'Hora, 2010). Before the training starts, the user conducts a pre-test of 55 questions in which relational ability is assessed. The result of this pre-test is a relational abilities score. Following this pre-test, training begins. In the training phase, users advance through 55 stages with increasing level of difficulty. On each level, the user is presented with a set of propositions including nonsense words, followed by a question in the following manner:

GYQ is the same as FYW
FYW is the same as VOP
Is VOP the same as GYQ?
YES (or) NO

The number of propositions and relations involved gradually becomes more difficult as training progresses. In the above example, two propositions involving a relation of coordination each (i.e., *same as*) were used. Additional frames taught in SMART are *opposite to*, *more than* and *less than*.

Each stage starts with a training block where auditory and visual feedback is given. When a training block is completed, the test block is administered without any feedback. After successfully completing a test block, the user then moves on to the next stage. When all 55 stages are completed, the user finally takes a second assessment of relational abilities. The recommended training amount is 30 minutes, two to three times per week, during approximately eight to twelve weeks (RaiseYourIQ, 2014).

Mathematical abilities. Mathematical abilities were operationalized as total number of correct responses on mathematical tests. Two similar mathematical tests constructed by the authors were used to measure mathematical abilities. Test items were inspired by Swedish educational literature (Holmström & Smedhamre, 2007). Some items were also influenced by previous Swedish national high school tests in mathematics (PRIM-gruppen, 2014). In order to further assure the face validity and representativeness of the tests, all test items were examined by a licensed high school teacher in mathematics. Items were then randomly distributed into two different but equivalent versions for pre-test and post-test (e.g., test 001 and test 002). The mathematical tests were randomly administered at pre-test and participants who were administered test 001 were administered test 002 at post-test and vice versa. The mathematical tests consisted of 44 items each, with a time limit of 30 minutes. For every item in the tests, participants could earn one (1) point (see Appendices A and B for a complete list of test items).

Logical reasoning and general intellectual ability. In order to assess general intellectual ability and skills related to logical reasoning and problem solving, Raven's Standard Progressive Matrices (SPM) was administered at pre-test and post-test. SPM consists of 60 items of visual-puzzle type tasks divided into five sets (e.g., A-E) with each set increasing in difficulty. The participant is presented with geometrical figures in which one part is missing. The task is to select the correct answer among a set of alternatives. For every item correctly answered, the participants earned one (1) point. SPM has good psychometrical properties with reported test-retest coefficients ranging from .80 to .90 over the course of one year, and validity studies indicate that SPM provides a good measure of general intellectual ability (see Raven, Raven, & Court, 2000).

Relational abilities. In order to assess whether SMART training affects relational abilities (e.g. arbitrarily applicable relational responses), a test of relational abilities is built into the SMART training program. The test consists of 55 questions that assess relational abilities according to the frames *same as*, *opposite to*, *less than* and *more than*. The questions are constructed like those employed in the SMART training (see description above). The result of this test is the RAI score. Relational abilities were assessed at pre-test and post-test. Previous studies indicate that a RAI score is a valid measure of relational abilities, and that RAI scores are related to traditional measures of intelligence (e.g., Cassidy, 2008; Cassidy, Roche, & Hayes, 2011). The test of relational abilities in the SMART training program also provides a measure of the total time needed to complete the 55 questions. Time based fluency (i.e., processing speed) is suggested to contribute to the performance variability of individuals on both everyday tasks and various psychological tests (Williams, Meyerson, & Hale, 2008). In the current study, this data together with number of correct responses is used to assess fluency of relational responding. The formula used to calculate RAI fluency was;

$$fluency = \left(\frac{x}{y}\right) \times 1000$$

where x is total RAI score and y is total time needed to complete the test of relational abilities. A higher number thus indicates a higher fluency (i.e., processing speed) on relational abilities.

Procedure

The authors visited different schools and classrooms to present the study. After presentation, notice of interest was collected and the students were then invited by e-mail to the pre-test sessions. The first pre-test session included a mathematical test and Raven's Standard Progressive Matrices. At the second pre-test session participants conducted a test of relational abilities (e.g., included in the SMART training program). A total of 19 students participated in the first round of pre-test sessions. For logistical reasons and to facilitate participation, the pre-test sessions were scheduled to fit the participants' school calendars and were therefore conducted at different days over the course of one week. Based on the date of completing the pre-test sessions, the participants were assigned to either experimental group with access to the SMART training program (n= 12), or control group (n= 7). Two participants assigned to the experimental group were (prior to training) transferred to the control group after reporting lack of time to participate in the training. One participant from the experimental group dropped out after the pre-test. After the first session, a total of 18 students participated in the study (experimental group n= 9, control group n= 9).

A second recruitment process was conducted in other high school classes in order to increase the sample size. The procedure was the same as in the first recruitment with two exceptions: (1) Pre-tests (e.g., mathematical test and Raven's Standard Progressive Matrices) were administered to all participants on the same occasion; (2) Due to technological constraints at the pre-test location, participants completed the computer-based test of relational ability unsupervised at location chosen by them. The completions of the computer-based pre-test were monitored online and some of the participants were contacted by phone or e-mail to ensure that the task was fully understood. Within a week, the pre-tests of the second group were completed. A total of 18 students completed the second pre-test session. Participants were then randomly assigned to either experimental group (*n*= 9) or control group (*n*= 8) and the experimental group was given access to the SMART training program.

Following pre-tests, the experimental group was given instructions for the SMART training. Training instructions were based on official guidelines for the SMART training program available online at www.raiseyouriq.com. After controlling for the necessary English skills, the experimental group was given access and started training. Training lasted for approximately eight to ten weeks. The experimental group was invited to weekly motivational sessions during the training period. Group and individual feedback and prompting were given via e-mails.

Post measures consisted of a mathematical test equivalent to the pre-test, Raven´s Standard Progressive Matrices, and the computer-based test of relational abilities. Post-test sessions were conducted directly following the completion of SMART training. Due to practical reasons, the post-tests were scheduled to fit the participants' school calendars and were therefore held over three occasions. Some participants (*n*= 8) conducted the post-RAI measurement at time and location of their own choice.

Ethical considerations

All participants under the age of 18 were required to provide a signed informed consent document from their caregivers. The informed consent document included contact information, information about the study, how data was to be handled and reported, and the opportunity to withdraw from the study at any time. All participants in the study also signed a document of individual informed consent prior to the pre-test session. After the pre-test and post-test sessions, short debriefings were offered. After the post-test session, all participants in the control group were offered two months access to the SMART training program. Participants were given unique ID's to guarantee anonymity. All experimental data were stored in a passcode protected fire-resistant safe. The current study was ethically reviewed and accepted at the Department of Psychology, Mid Sweden University, as a part of the first authors' master thesis.

Results

Eight participants from the experimental group were excluded from the analyses due to not participating in the post testing (*n*= 3), failing to follow instructions given at both pre-tests and post-tests (*n*= 1), or due to low rates of SMART training (*n*= 4). Six participants from the control group were excluded from the analyses due to not participating in post-tests. The final sample consisted of 21 participants (experimental group *n*= 10, control group *n*= 11).

Independent samples *t*-tests showed a significant difference between the groups, with the experimental group scoring higher than the control group on pre-mathematical performance (*t*(19)= 2.11, *p*= .048, *M*= 31.20 vs. 24.64, *CI* [.063, 13.065]). The experimental group had slightly higher pre-Raven scores, but the difference was not significant (*t*(19)= 1.06, *p*= .29, *M*= 51.10 vs. 48.55, *CI* [-1.012, 12.721]). Also, there was a marginally significant difference between the groups with respect to age (*t*(19)= 2.02, *p*= .058), due to the experimental being older. Age was also marginally significantly correlated with the pre-mathematical ability scores (*r*= .42, *p*= .061,), and pre-Raven scores (*r*= .43, *p*= .052).

When analyzing levels of mathematical performance and scores on Raven, Analysis of Covariance (ANCOVA) was used to index change from pre-test to post-test, while simultaneously controlling for pre-intervention levels. Due to an almost significant difference with respect to age between the groups and its association to the pre-test measures, age was included as a covariate in the analyses.

The 2 (Condition: experimental vs. control) x 2 (Time: pre vs. post) ANOVA showed, as expected, a significant Group X Time interaction effect (*F*(1, 19)= 5.94, *p*= .025) on RAI accuracy (% correct responses) and RAI fluency (*F*(1, 19)= 8.22, *p*= .010). Paired samples *t*-tests showed a significant increase from pre-test to post-test on RAI accuracy (*t*(9)= 6.07, *p* <.001, *M*= 44.00 vs. 52.40, *CI* [5.269, 11.531], *d*= 1.92) and RAI fluency scores (*t*(9)= 4.19, *p*= .002, *M*= 49.57 vs. 70.62, *CI* [13.364, 44.710], *d*= 1.33) for the experimental group, but no significant increase from pre-test to post-test on either RAI accuracy (*t*(10)= .93, *p*= .38, *M*= 43.09 vs. 45.09, *CI* [-2.816, 6.816], *d*= .29) or RAI fluency scores (*t*(10)= 1.81, *p*= .10, *M*= 52.28 vs. 59.19, *CI* [-1.594, 15.419], *d*= .55) for the control group.

The ANCOVA that was used to index change from pre-test to post-test revealed a significant difference between the groups in post-Raven's total score (*F*(1, 17)= 5.46, *p*= .032, η_p^2= .24), estimated marginal means= 52.26 points vs. 49.94 points, for experimental and control respectively).

When repeating the ANCOVA using stricter inclusion criteria (i.e., completion of all 55 stages included in the SMART training program among experimental participants; see Table 1 for descriptive data for each participant), a significant difference between the groups was again observed (*F*(1, 12)= 7.18, *p*= .020, η_p^2= .37, estimated marginal means= 52.98 points vs. 49.74 points, for experimental and control respectively). The fact that the significant difference in post-Raven's total score remained significant despite substantially less participants in the experimental group suggests that the amount of practice increases the magnitude of effects.

The ANCOVA that was used to index change from pre-test to post-test did not find a significant difference between the two groups on post-mathematical performance (*F*(1, 17)= .64, *p*= .44, η_p^2= .036, estimated marginal means= 29.61 points vs. estimated marginal means= 31.00 points, for experimental and control respectively), but a main effect of Age (*F*(1, 17)= .6.13, *p*= .024, η_p^2= .27), with increased age being associated with better performance.

When repeating the ANCOVA using stricter inclusion criteria (i.e., completion of all 55 stages included in the SMART training program among experimental participants; see Table 1 for descriptive data for each participant), this analysis did not change the pattern (*F*(1, 12)= .49, *p*= .50, η_p^2= .039).

Table 1. Participant descriptives, total scores, means (*M*), and standard deviations (*SD*) on outcome variables.

Partic.	Age	SSC	ML	Pr-RAI	Po-RAI	PrM	PoM	PrR	PoR	PrF	PoF
E01	206	21	2	54	55	33	37	57	59	63.83	65.01
E02	210	26	2	43	55	39	40	46	48	51.01	65.24
E03	211	55	2	49	54	40	36	55	58	60.42	66.58
E04	211	20	3	38	49	25	28	48	48	36.43	59.39
E05	200	29	2	48	52	34	33	51	51	56.60	77.84
E06	214	21	2	39	46	18	22	49	54	39.24	53.80
E07	215	55	2	47	55	39	41	57	57	57.60	118.79
E08	217	55	2	36	52	33	38	54	55	38.14	79.15
E09	207	55	2	40	51	29	32	44	49	47.62	107.82
E10	216	55	2	46	55	22	27	50	53	44.80	92.44
M	210.7	39.2	N.a.	44	52.4	31.2	33.4	51.1	53.2	49.57	78.61
SD	5.25	16.86	N.a.	5.74	3.06	7.57	6.19	4.53	4.10	9.85	21.48
C01	209	N.a.	2	39	42	18	25	54	54	55.32	64.42
C02	201	N.a.	1	49	36	27	22	43	46	77.41	61.22
C03	189	N.a.	1	42	39	19	16	39	46	60.70	46.43
C04	203	N.a.	1	43	42	25	25	50	48	40.68	43.57
C05	224	N.a.	1	44	53	34	38	51	51	48.94	76.37
C06	198	N.a.	1	28	43	13	14	40	41	34.48	52.83
C07	195	N.a.	1	40	41	18	23	48	46	41.75	52.90
C08	201	N.a.	1	51	52	32	33	55	57	61.81	72.83
C09	215	N.a.	2	45	51	29	35	47	47	48.28	58.02
C10	205	N.a.	1	46	45	28	38	58	55	52.33	60.08
C11	202	N.a.	1	47	52	28	34	49	49	53.35	62.42
M	203.82	N.a.	N.a.	43.09	45.09	24.64	27.55	48.55	49.09	52.28	59.19
SD	9.55	N.a.	N.a.	6.17	5.94	6.67	8.53	6.06	4.74	11.80	10.06

Notes: Partic.= participant and group (E= experimental group; C= control group); Age= participant age in months; SSC= SMART program training stages completed; ML= mathematical course level; PrRAI= pre-RAI total score; PoRAI= post-RAI total score; PrM= pre-mathematical test total score; PoM= post-mathematical test total score; PrR= pre-Raven's total score; PoR= post-Raven's total score; PrF= pre-RAI fluency score; PoF= post-RAI fluency score; N.a.= Not applicable.

DISCUSSION

The main purpose of the present study was to investigate if multiple exemplar training of derived relational responding in accordance with the relational frames same as, opposite to, more than and less than, would increase mathematical skills in a sample of Swedish high school students. A secondary purpose of the present study was to explore the effects of relational training on cognitive abilities as measured by Raven's Standard Progressive Matrices.

As expected, and in accordance with previous research (see for example Cassidy, 2008), the MET as employed in the SMART training program was found to increase RAI accuracy scores and RAI fluency scores in the experimental group, an increase that was absent for the control group. Given the positive correlation between relational abilities and intelligence (Cassidy, 2008; Cassidy et al., 2011; O'Hora et al., 2005), these findings should predict an increase in IQ as measured by the SPM. Results showed a significant difference between the groups on SPM performance (i.e., general intellectual ability), and this difference remained significant after controlling for age and pre-test performance on SPM. In addition, when comparing the SPM scores among those participants who fully completed the SMART training (i.e., completed all 55 stages) with the control group, the magnitude of effects was even stronger. It therefore appears that the amount of training is important, as suggested by RaiseYourIQ (2014). Previous research has shown a correlation of .74 between SPM and full scale IQ for the age group in the current study (O'Leary, Rusch, & Guastello, 1991), which suggests that the participants in the experimental group increased their full scale IQs.

More importantly, when analyses were performed on post-mathematical test scores, no significant difference was found between the groups on post-mathematical test scores. It should be noted that, even though there was a significant difference between the groups on pre-mathematical test scores as well as a difference in age, the absence of a significant difference between groups on post-mathematical test scores was not a function of pre-mathematical test scores or age. In addition, it is important to recognize that absence of a significant effect following training was not a function of an ineffective intervention, nor a low sample size, as the experimental intervention was in fact effective in enhancing relational framing as evident on a measure of intelligence. Also, even though none of the students had English as their first language, they were sufficiently proficient in English to work with the relatively simple frame words (same as, opposite to, more than, less than).

There is a possibility that the relational frames trained in the SMART program (e.g., frames of comparison, coordination and opposition) are insufficient to improve mathematical performance in a high school population, and that training of more complex relational tasks that involve other frames is required in order to improve mathematical performance for the current population.

For the aim of the current study, using the SMART training program was beneficial for both scientific and practical reasons. First, SMART provided data on pre- and post-relational abilities as well as an overview of the ongoing training process for each individual participant in the study (e.g., stages completed, amount of questions answered etc.). The opportunity to monitor the participant's training progress facilitated individual prompting and reinforcement for further training. Second, the fact that SMART is an online training tool made it highly accessible for the participants as the training could be performed on computer or tablet, at locations and hour of day as chosen by the participants. However, lack of adherence seems to be a general problem in many Internet based interventions (see for example, Waller & Gilbody, 2009). Despite prompts and written feedback given by the authors, only 5 participants completed the entire SMART training program according to the instructions based on the recommendations from RaiseYourIQ (2014). The insufficient rate of training in the current study can have several explanations. It is reasonable to believe that our participants are exposed to other competing contingencies in their everyday lives, with school demands being one such possible factor, and therefore making it more difficult to adhere to the training recommendations. Also, the SMART program layout may not appeal to the current population. In the SMART program, users can choose between child and adult layouts, however the difference in layout is negligible. Vivid colors and cartoon characters characterize the SMART program and the overall layout appears to be configured to suit younger populations. A review of adherence to Internet based interventions by Kelders, Kok, Ossebaard, and Van Gemert-Pijnen (2012) suggests that design is an important factor to consider regarding adherence to online interventions. The SMART training is also fairly monotonous which may further have served to reduce adherence.

It is clear that MET is an effective intervention to enhance relational framing, even though the majority of research is restricted to younger populations or populations with learning difficulties (see for example Stewart, Tarbox, Roche, & O'Hora, 2013). The cumulative nature of relational framing is not yet fully understood (Barnes-Holmes, Foody, Barnes-Holmes, & McHugh, 2013), and therefore it might be the case that populations with specific goals require more tailored interventions, which involve different frames and complexity of the relational tasks.

In summary, even though the current study failed to provide evidence for the efficacy of relational training on mathematical skills, it adds to the growing body of research on RFT and intellectual performance. The SMART training program proved yet again to have an effect on a psychometrically validated and well-recognized test of general intellectual ability other than the Wechsler scales. This strengthens the validity of using behaviorally based interventions to improve intellectual abilities. In addition, the current study showed that the effect of SMART training on general intellectual ability is related to the amount of training, which supports the notion that exposure to multiple exemplars is a key feature for developing the relational skills underlying intellectual performance. Also, the participants in the present study differed regarding age and setting from those in previous research (see for example Cassidy, 2008; Cassidy et al., 2011), which indicates that behaviorally based interventions used to improve intellectual abilities with children can be generalized to other populations.

References

Anderson CM & Kincaid D (2005). Applying behavior analysis to school violence and discipline problems: Schoolwide positive behavior support. *The Behavior Analyst, 28*, 49-63.

Barnes-Holmes Y, Barnes-Holmes D, & Cullinan V (2001) Education. In SC Hayes, D Barnes-Holmes, & B Roche (Eds.), *Relational frame theory, a post-Skinnerian account of human language and cognition* (pp. 181-195). New York: Plenum Publishing Co.

Barnes-Holmes Y, Foody M, Barnes-Holmes D, & McHugh L (2013). Advances in research on deictic relations and perspective-taking. In S Dymond & B Roche (Eds), *Advances in relational frame theory-research and application* (pp. 127-148). Oakland, CA: New Harbinger Publications, Inc.

Butterworth B, Varm S, & Laurillard D (2011). Dyscalculia: From brain to education. *Science, 332*, 1049-1053. DOI:10.1126/science.1201536

Butterworth B (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46*, 3-18. DOI: 10.1111/j.1469-7610.2004.00374.x

Cassidy S, Roche B, & Hayes SC (2011). A relational frame training intervention to raise intelligence quotients: A pilot study. *The Psychological Record, 61*, 173-198.

Cassidy S, Roche BT, & O'Hora D (2010). Relational frame theory and intelligence. *European Journal of Behaviour Analysis, 11,* 37-52. DOI: 10.1080/15021149.2010.11434333

Cassidy S (2008). *Relational frame theory and human intelligence: A conceptual and empirical analysis*. Doctoral thesis. The National University of Ireland, Maynooth. Available: eprints.nuim.ie/1489/1/sarahfinalphd.pdf

Daly EJ III, Garbacz SA, Olson SC, Persampieri M, & Ni, H (2006). Improving oral reading fluency by influencing students' choice of instructional procedures: An experimental analysis with two students with behavioral disorders. *Behavioral Interventions, 21*, 13-30. DOI: 10.1002/bin.208

DeStefano D & LeFevre J (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16*, 353-386. DOI: 10.1080/09541440244000328

Duncan GJ, Dowsett CJ, Claessens A, Magnuson K, Huston AC, Klebanov P, Pagani LS, Feinstein L, Engel M, Brooks-Gunn J, Sexton H, Duckworth K, & Japel C (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428-1446. DOI: 10.1037/0012-1649.43.6.1428

Eckert TL, Ardoin SP, Daly EJ III, & Martens BK (2002). Improving oral reading fluency: A brief experimental analysis of combining an antecedent intervention with consequences. *Journal of Applied Behavior Analysis, 35*, 271-281. DOI: 10.1901/jaba.2002.35-271

Fienup DM, Covey DP, & Critchfield TS (2010). Teaching, brain-behavior relations economically with stimulus equivalence technology. *Journal of Applied Behavior Analysis, 43*, 19-33. DOI: 10.1901/jaba.2010.43-19

Gross AC & Fox EJ (2009). Relational frame theory: An overview of the controversy. *Analysis of Verbal Behavior, 25*, 87-98.

Hayes SC, Barnes-Holmes D, & Roche B (2001). *Relational frame theory, a post-Skinnerian account of human language and cognition*. New York: Plenum Publishing Co.

Hayes SC, Blackledge JT, & Barnes-Holmes D (2001). Language and cognition: constructing and an alternative approach within the behavioral tradition. In SC Hayes, D Barnes-Holmes, & B Roche (Eds.), *Relational frame theory, a post-Skinnerian account of human language and cognition* (pp. 3-20). New York: Plenum Publishing Co.

Hayes SC, Fox E, Gifford EV, Wilson KG, Barnes-Holmes D, & Healy O (2001). Derived relational responding as learned behavior. In SC Hayes, D Barnes-Holmes, & B Roche (Eds.), *Relational frame theory, a post-Skinnerian account of human language and cognition* (pp. 21-49). New York: Plenum Publishing Co.

Hayes SC & Hayes LJ (1992). Verbal relations and the evolution of behavior analysis. *American Psychologist, 47*, 1383-1395. DOI: 10.1037/0003-066X.47.11.1383

Hitch GJ & Baddeley AD (1976). Verbal reasoning and working memory. *The Quarterly Journal of Experimental Psychology, 28*, 603-621. DOI: 10.1080/14640747608400587

Holmes J, Gathercole SE, & Dunning DL (2009). Adaptive training leads to sustained enhancement of poor working memory in children. *Developmental Science, 12*, 9-15. DOI: 10.1111/j.1467-7687.2009.00848.x

Holmström M & Smedhamre E (2007). *Matematik för gymnasiet kurs A*. Stockholm: Liber.

Kratochwill TR & Martens BK (1994). Applied behavior analysis and school psychology. *Journal of Applied Behavior Analysis, 27*, 3-5. DOI: 10.1901/jaba.1994.27-3

Kelders SM, Kok RN, Ossebaard HC, & Van Gemert-Pijnen JEWC (2012). Persuasive system design does matter: A systematic review of adherence to web-based interventions. *Journal of Medical Internet Research, 14*, 16-39. DOI: 10.2196/jmir.2104

Livne NL & Milgram RM. (2006). Academic versus creative abilities in mathematics: Two components of the same construct? *Creativity Research Journal, 18*, 199-212. DOI: 10.1207/s15326934crj1802_6

Martens BK & Ardoin SP (2002). Training school psychologists in behavior support consultation. *Child & Family Behavior Therapy, 24*(1-2), 147-163. doi: 10.1300/J019v24n01_10

Melby-Lervåg M & Hulme C (2013). Is working memory training effective? A meta-analytic review. *Developmental Psychology, 49*, 270-291. DOI: 10.1037/a0028228

Meyer KA (1999). Functional analysis and treatment of problem behavior exhibited by elementary school children. *Journal of Applied Behavior Analysis, 32*, 229-232. DOI: 10.1901/jaba.1999.32-229

Miller DL & Kelley ML (1994). The use of goal setting and contingency contracting for improving children's homework performance. *Journal of Applied Behavior Analysis, 27*, 73-84. DOI: 10.1901/jaba.1994.27-73

Ninness C, Rumph R, McCuller G, Harrison C, Ford AM, & Ninness SK (2005). A functional analytic approach to computer-interactive mathematics. *Journal of Applied Behavior Analysis, 38*, 1-22. DOI: 10.1901/jaba.2005.2-04

O'Hora D, Peláez M, & Barnes-Holmes D (2005). Derived relational responding and performance on verbal subtests of the WAIS-III. *The Psychological Record, 55*, 155-175.

O'Leary U, Rusch KM, & Guastello, SJ (1991). Estimating age-stratified WAIS-R IQs from scores on the raven's standard progressive matrices. *Journal of Clinical Psychology, 47*, 277-284.

PRIM-gruppen (2014). *Previously administered high school national tests of arithmetics*. Retrieved from http://www.su.se/primgruppen/matematik/kurs-1/tidigare-prov

RaiseYourIQ (2014). *SMART training program*. Retrieved from http://www.raiseyouriq.com

Raven J, Raven JC, & Court JH (2000). *Manual for Raven's Progressive Matrices and Vocabulary scales*. Section 3: The Standard Progressive Matrices. San Antonio, TX: Pearson, Inc.

Sidman, M. (1971). Reading and auditory-visual equivalences. *Journal of Speech, Language, and Hearing Research*, *14,* 5-13. DOI: 10.1044/jshr.1401.05

Skinner BF (1957). *Verbal behavior.* Englewood Cliffs, NJ: Prentice-Hall.

Stewart I & Roche B (2013). Relational frame theory: An overview. In S Dymond & B Roche (Eds.), *Advances in relational frame theory-research and application* (pp. 51-71). Oakland, CA: New Harbinger Publications, Inc.

Stewart I, Tarbox J, Roche B, & O'Hora D (2013). Education, intellectual development, and relational frame theory. In S Dymond & B Roche (Eds.), *Advances in relational frame theory-research and application* (pp. 177-197). Oakland, CA: New Harbinger Publications, Inc.

Stone JE (1994). Developmentalism's impediments to school reform: Three recommendations for overcoming them. In R Gardner, DM Sainato, Cooper JO, Heron TE, Heward WL, Eshleman J, & Grossi TA (Eds.), *Behavior Analysis in Education: Focus on Measurably Superior Instruction* (pp. 57-72). Pacific Grove, CA: Brooks Cole Publishing.

Waller R & Gilbody S (2009). Barriers to the uptake of computerized cognitive behavioural therapy: A systematic review of the quantitative and qualitative evidence. *Psychological Medicine, 39*, 705-712. DOI: 10.1017/S0033291708004224

Wechsler D (2003). *Wechsler Intelligence Scale for Children-Fourth Edition. Technical and Interpretive Manual*. London: Pearson, Inc.

Williams B, Meyerson J, & Hale, S. (2008). Individual differences, intelligence, and behavior analysis. *Journal of the Experimental Analysis of Behavior*, *90*, 219-231.

Received, September 23, 2015
Final Acceptance, March 30, 2016